

Preface

It is a great pleasure to introduce this special volume of the *Polynesian Journal of Mathematics*, dedicated to the mathematics of René Schoof.

The volume has its origins in René 25, a conference held at the University of French Polynesia in Tahiti from 18 to 22 August 2025 to celebrate René Schoof and his wide-ranging contributions to mathematics. The conference brought together researchers from many parts of the world whose work is connected, directly or indirectly, with themes that have played a central role in René's research: algebraic number theory, arithmetic geometry, computational number theory, elliptic curves, abelian varieties, and coding theory. The present volume collects a selection of contributions related to the mathematics presented and discussed on that occasion, together with other contributions inspired by René's work.

A volume in René's honour is particularly appropriate because it is difficult to associate his name with a single area of mathematics. His work has repeatedly crossed the boundaries between different parts of number theory and arithmetic geometry. What connects many of his contributions is perhaps less a particular subject than a distinctive mathematical philosophy: the conviction that deep arithmetic questions can often be understood, and sometimes solved, by combining structural ideas with explicit computation.

This combination is already visible in some of René's earliest and best-known work. Starting from his first major result, the algorithm for computing the number of points of an elliptic curve over a finite field, algorithm that still bears his name, and continuing to his contributions to the arithmetic of elliptic curves and modular forms, to the computation of class groups and zeta functions, to the arithmetic of abelian varieties, and to questions concerning finite flat group schemes have had a lasting influence. At the same time, his work has repeatedly demonstrated that algorithms are not merely tools for numerical experimentation. In the right hands, computation can reveal structure, suggest conjectures, and provide new ways of approaching questions that at first appear inaccessible.

The word *explicit* is therefore particularly important when describing René's mathematics. He has an unusual ability to move comfortably between abstract arithmetic geometry and concrete calculations. A theorem is often accompanied by a method for actually finding the objects whose existence it asserts; conversely, a computational question is rarely treated as merely computational, but becomes an entry point into a deeper arithmetic problem. This interplay between theory and computation has become increasingly important in modern number theory, and René has been one of the mathematicians who helped establish it as a natural and powerful way of doing mathematics.

The title of this volume, *AJAX — Arithmetic, Jacobians, Algorithms & Exposition*, was chosen with this breadth in mind. The first three words point to themes that recur throughout René's mathematical work, while the fourth is equally significant. René has always had a strong sense of exposition. His mathematics is often characterized by a remarkable economy of ideas: complicated arguments are reduced to their essential ingredients, and sophisticated theories are made concrete without sacrificing their depth.

The ability to explain difficult mathematics clearly is itself a considerable mathematical achievement, and it is one of the qualities that many of us associate with René.

The René 25 conference in Tahiti provided a particularly fitting setting for celebrating these aspects of his work. The location was, of course, extraordinary, but the real success of the meeting was the mathematical community that came together there. The programme reflected the remarkable breadth of René's influence, with talks ranging from modular forms, elliptic curves and abelian varieties to class groups, algorithms, isogenies, coding theory and arithmetic questions in positive characteristic. The list of participants also reflected the international nature of the community that René has helped to build over many decades.

The atmosphere of the conference was also important. A meeting held in honour of a mathematician who has been active for so many years inevitably brings together several generations of researchers. Some participants have collaborated with René, some have been influenced by his papers, and others work on subjects that have developed in directions that his ideas helped make possible. Such meetings are a reminder that the influence of a mathematician cannot be measured simply by the number of theorems bearing their name. It is also visible in the questions people ask, the methods they use, and the mathematical communities that grow around their ideas.

It is therefore a particular pleasure for us, to present this volume to the mathematical community. We hope that the articles collected here will not only contribute new mathematics, but will also convey something of the breadth of subjects, ideas and techniques to which René has contributed. The volume is not intended to provide a comprehensive account of his work—that would be neither possible nor appropriate—but rather to offer a collection of mathematical contributions that reflects some of the many directions in which his influence continues to be felt.

We would like to thank all the authors who contributed to the volume, as well as the referees who generously gave their time and expertise. We are grateful to Gaetan Bisson and Roger Oyono, who organized René 25, and to the members of the scientific committee for helping to create such a memorable meeting. We also thank the *Polynesian Journal of Mathematics* for providing a home for this collection and for supporting the idea of a special volume devoted to René's mathematics.

Finally, and most importantly, we would like to thank René himself—not only for the mathematics that inspired this volume, but for the friendship and companionship that have accompanied it. We hope that this volume will serve as a small expression of the esteem and affection in which René is held by his colleagues, his students, and his many friends around the world.

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